Parallelizing FFT with a Focus on Bandwidth Efficiency

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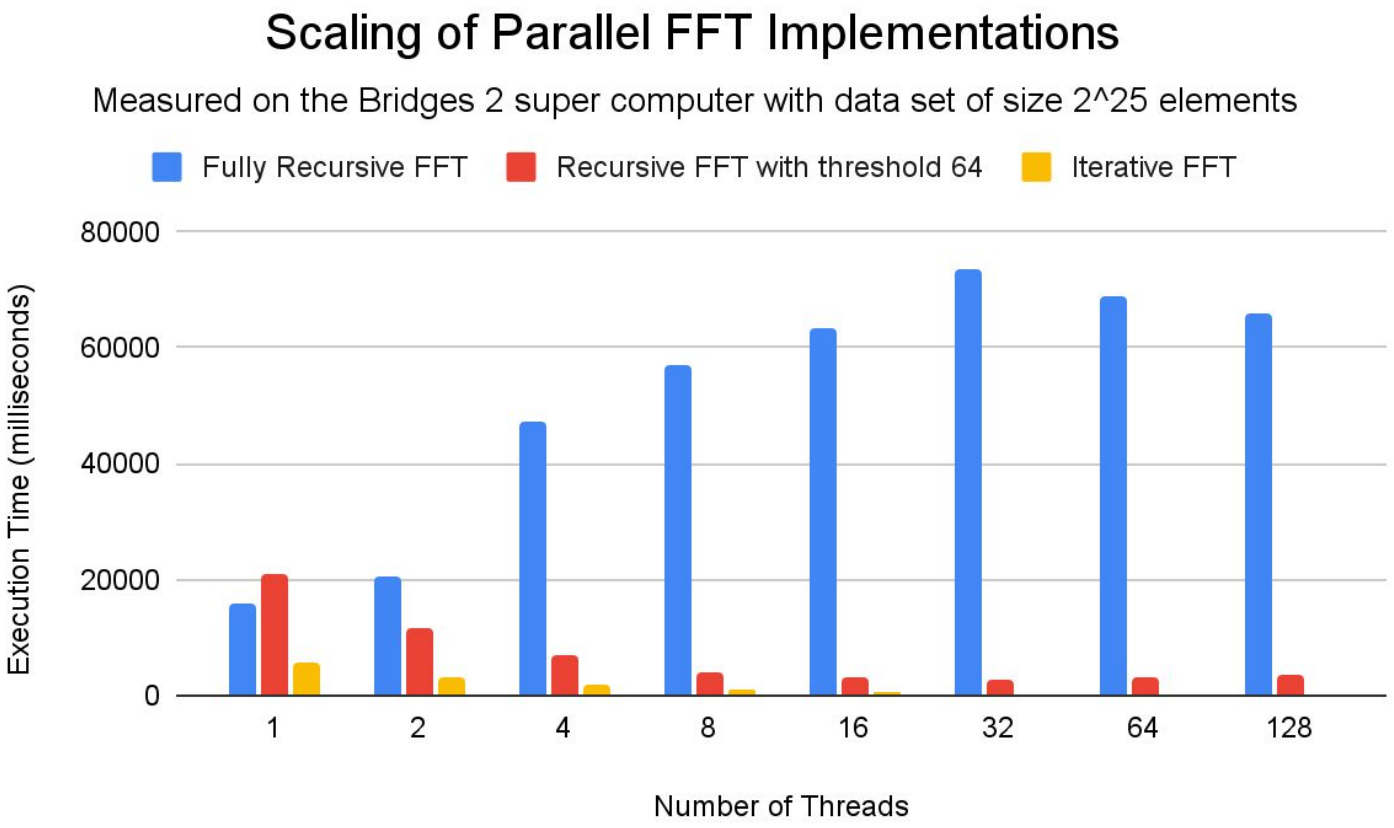
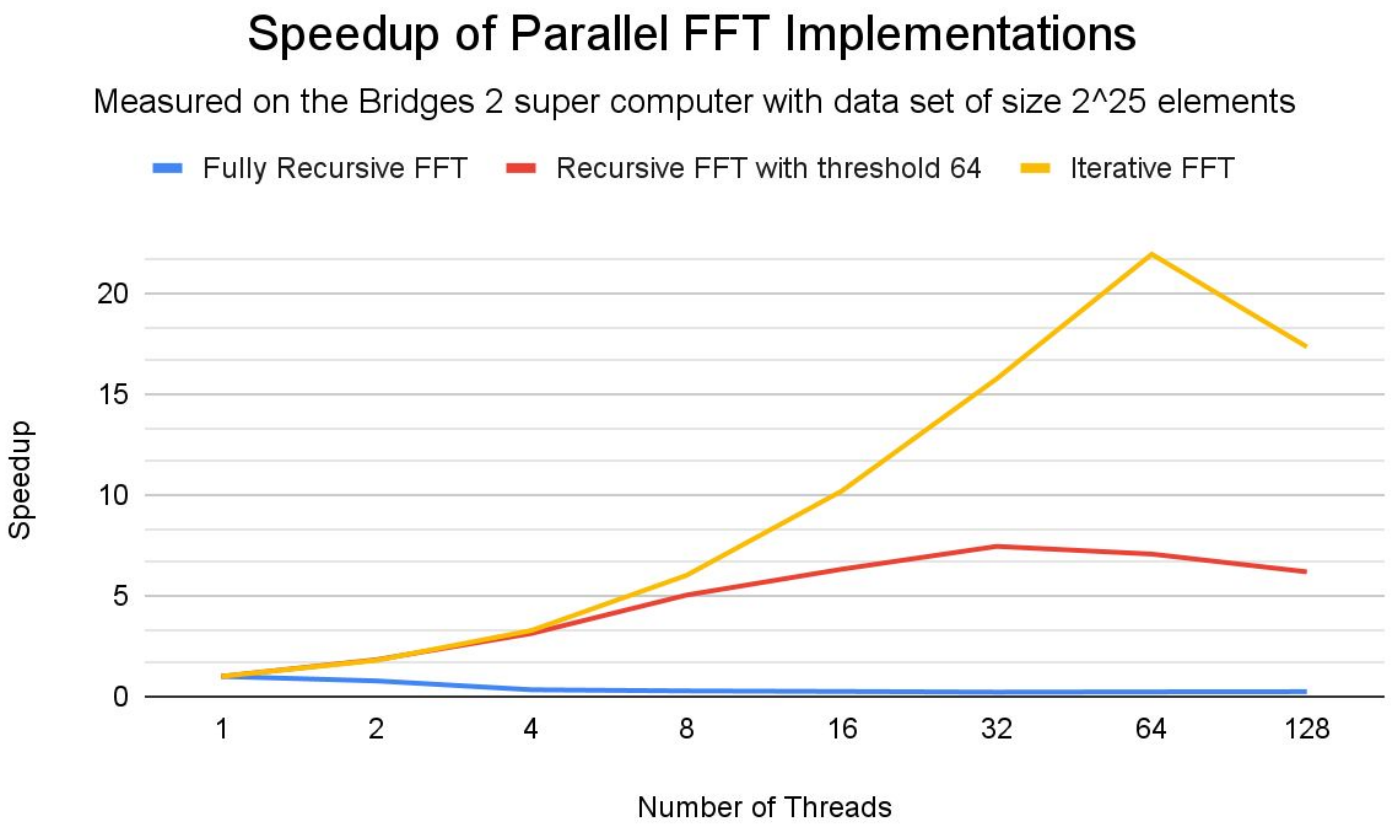
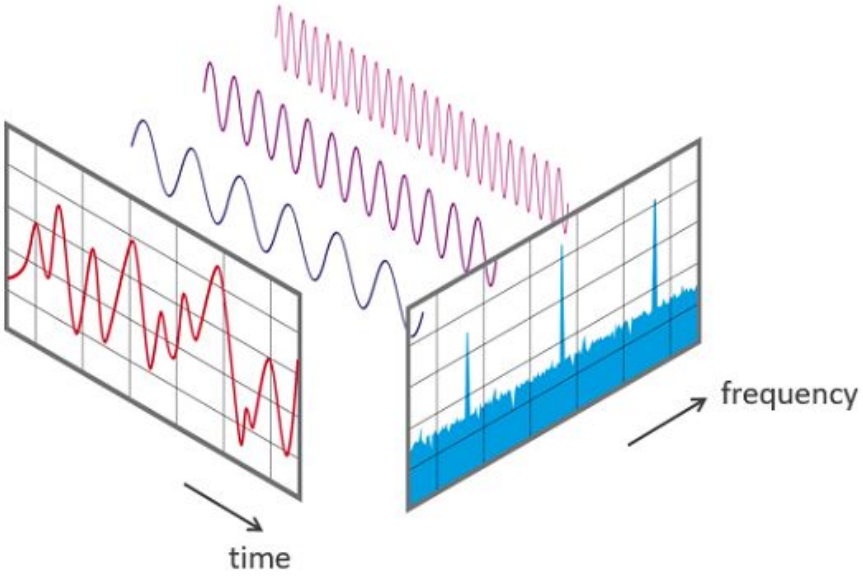




FFT and our Approach

- What is Fast Fourier Transform (FFT)?
- Discrete Fourier Transform
- FFT (Fully Recursive, Recursive with Threshold, and Iterative)
- 2D FFT
- Image Compression

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk}$$

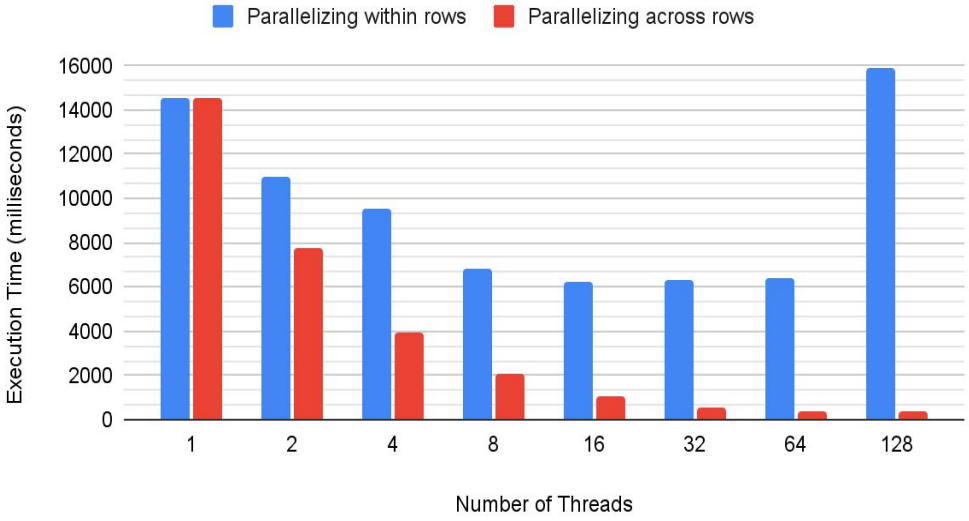


Main Optimizations

- Parallel pre-computation of Twiddle Factors
  - Loop collapse
- Switching to DFT in Recursive FFT at Threshold
- Iterative Algorithmic Modifications
- Bit Reversal allowing Butterfly Networks
- Chunking of Data that fits in L1/L2 Cache
- Matrix Transpose enabling better Spatial Locality
- Exploiting more available parallelism

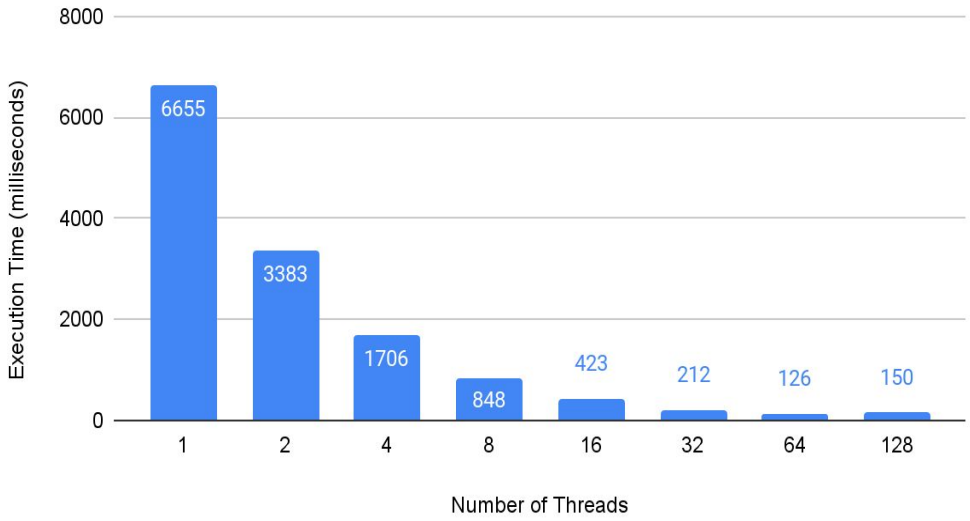
Scaling of 2D Parallel FFT

Measured on the Bridges 2 super computer with data set of size 2^14 x 2^14 elements



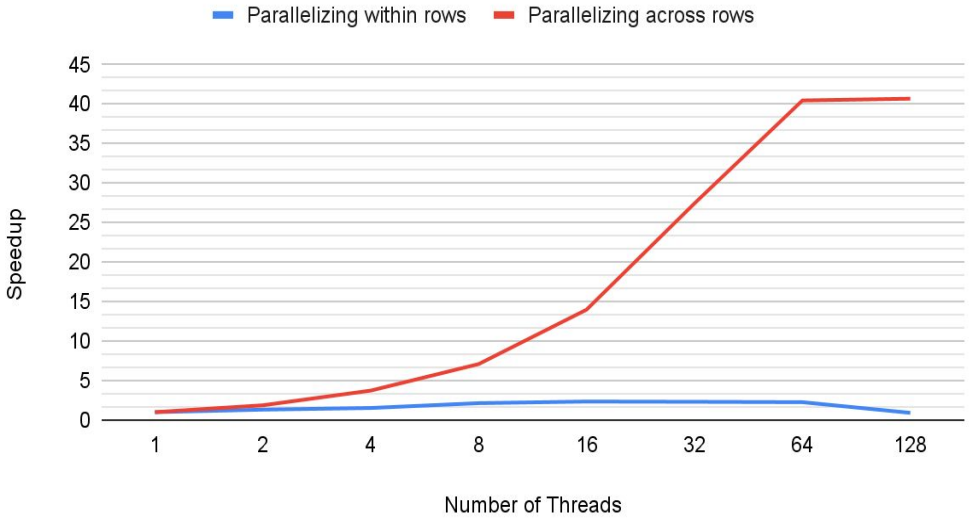
Scaling of DFT

Measured on the Bridges 2 super computer with data set of size 2^15 elements



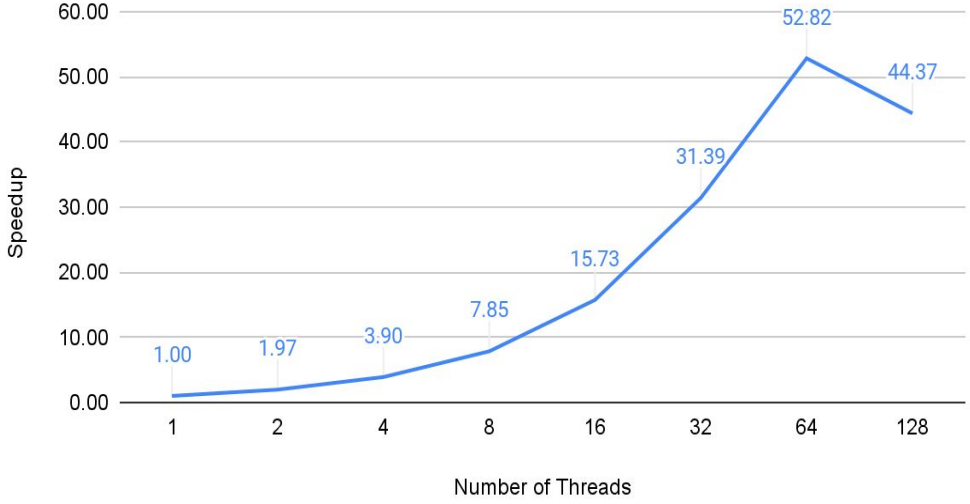
Speedup of 2D Parallel FFT

Measured on the Bridges 2 super computer with data set of size 2^14 x 2^14 elements



Speedup of DFT

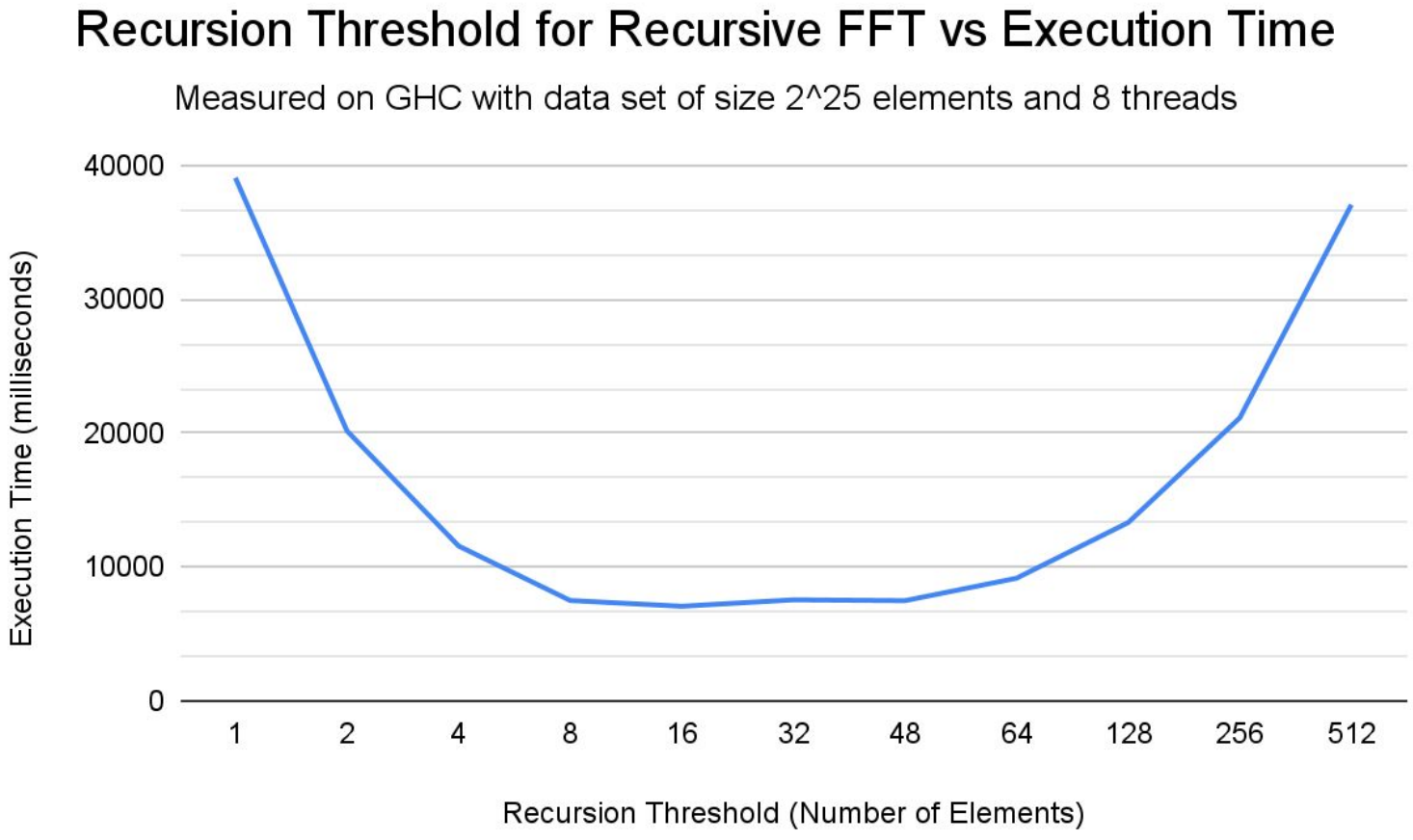
Measured on the Bridges 2 super computer with data set of size 2^15 elements

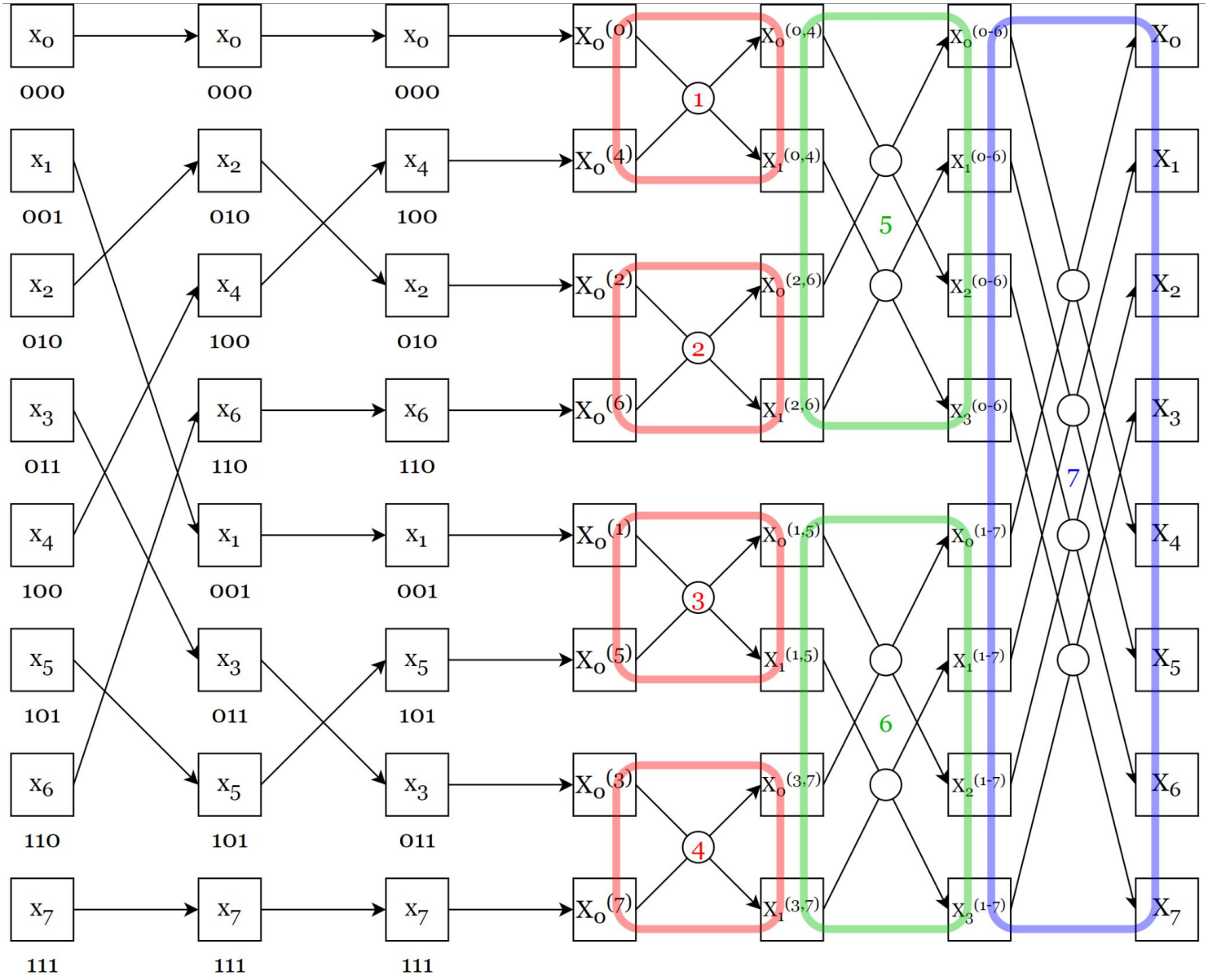


$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of even-indexed part of } x_n} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of odd-indexed part of } x_n}$$

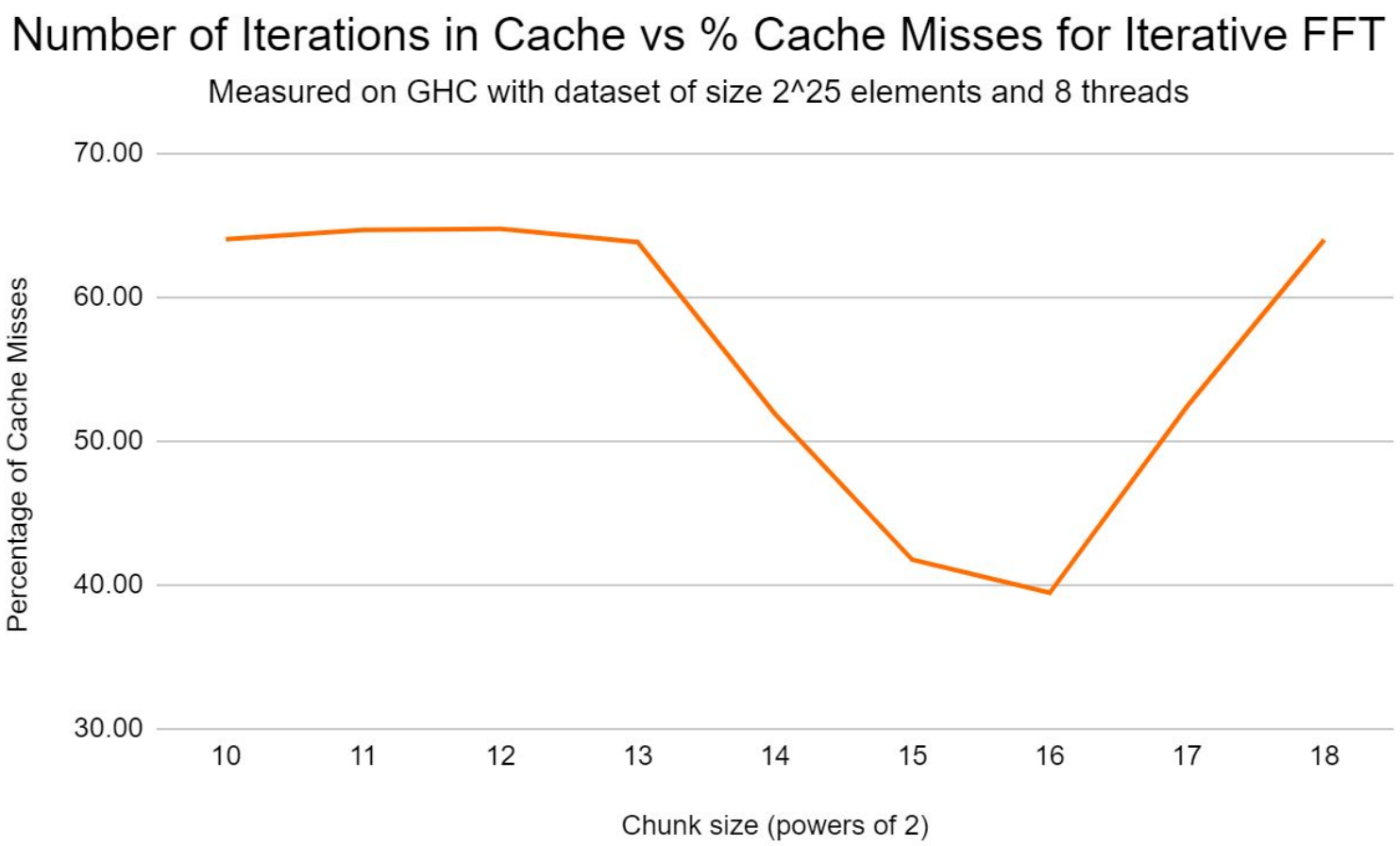
$$\begin{aligned} X_k &= E_k + e^{-\frac{2\pi i}{N} k} O_k \\ X_{k+\frac{N}{2}} &= E_k - e^{-\frac{2\pi i}{N} k} O_k \end{aligned}$$

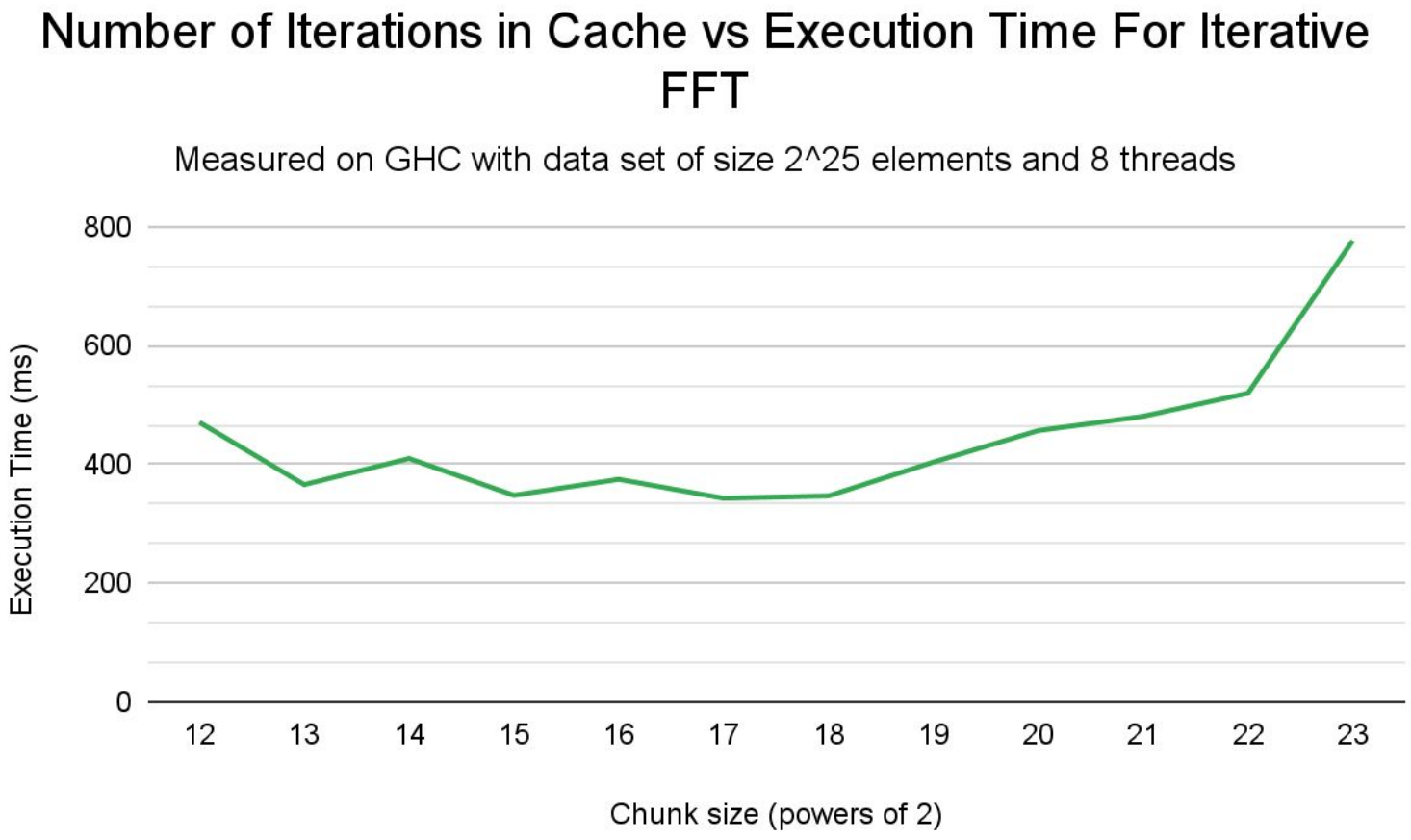
- Sequential access within a recursive call
- Needless copying causes poor locality
- High overhead due to recursion at small sizes
- Better scaling and constant factors for DFT

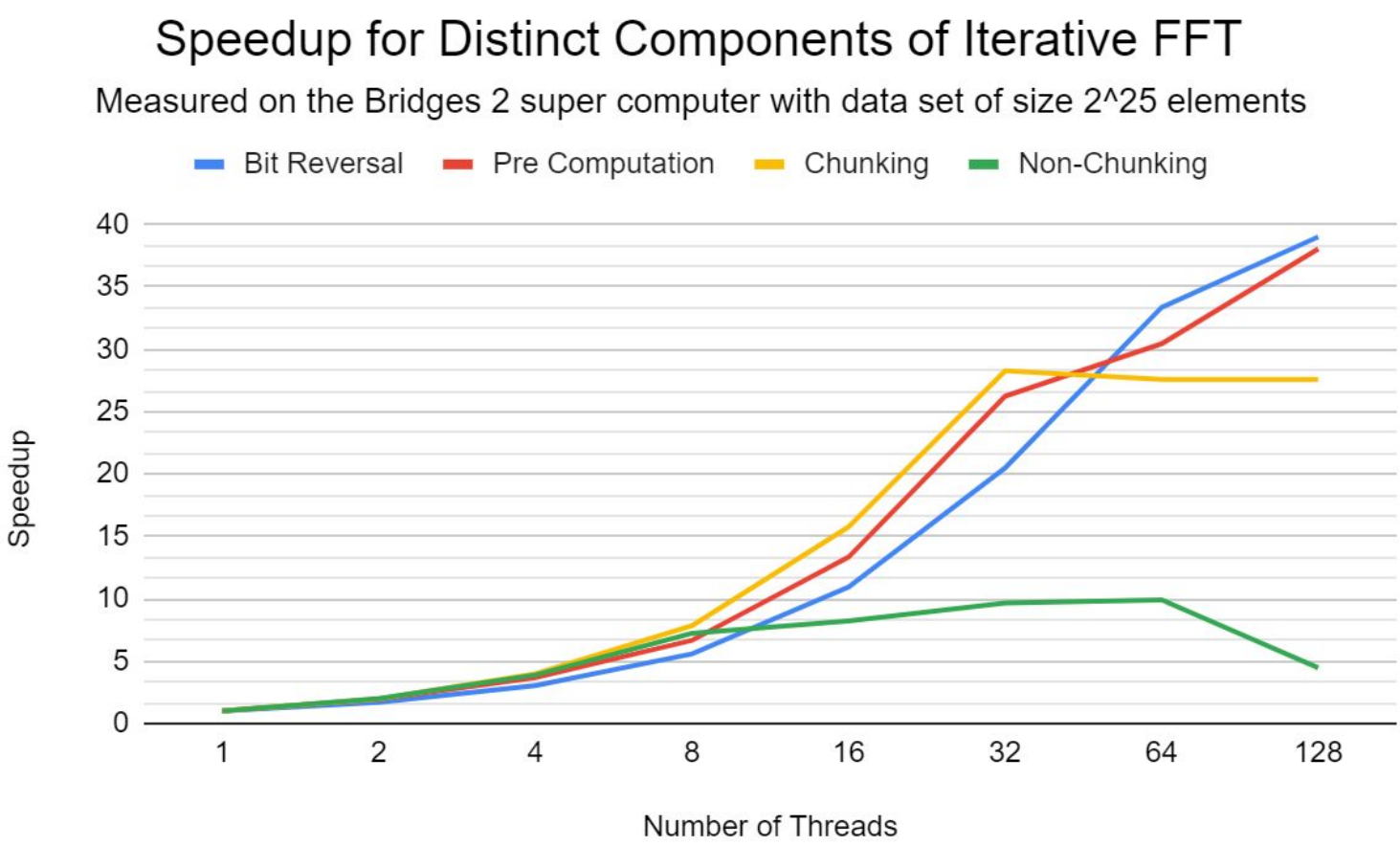




- Recursive Shuffle
  - Sort by reverse of the bits of the index
  - Bit reversal is a bijection and swaps are independent of each other
- Chunking (4 elements in cache)
  - 1 -> 2 -> 3 -> 4 -> 5 -> 6 -> 7
  - 5 reuses data brought in by 1 and 2
  - 1 -> 2 -> 5 -> 3 -> 4 -> 6 -> 7
  - N log(N/C) instead of N log(N)


Number of Iterations in Cache vs % Cache Misses for Iterative FFT
Measured on GHC with dataset of size 2^25 elements and 8 threads
70.00
60.00
50.00
40.00
30.00
Percentage of Cache Misses
10
11
12
13
14
15
16
17
18
Chunk size (powers of 2)


Number of Iterations in Cache vs Execution Time For Iterative FFT
Measured on GHC with data set of size 2^25 elements and 8 threads
800
600
400
200
0
Execution Time (ms)
12
13
14
15
16
17
18
19
20
21
22
23
Chunk size (powers of 2)



- Chunking is effective
- Non-chunking still bandwidth bound
- Bit reversal (bit shifting) and pre computation (std::polar) are more compute bound

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2-Dimensional FFT

- Steps
  - Perform in-place 1D FFT on the rows
  - Transpose the matrix
  - Perform in-place 1D FFT on the rows (which were the columns pre-transpose)
  - Transpose the matrix
- Parallel across rows or within row

Image Compression

- 2D FFT on each color obtaining the Fourier Coefficients
- Discard smallest x% of coefficients by magnitude
- Store remaining coefficients and their indices
- Convert back to matrix by adding zeros for missing data
- Perform Inverse FFT on the matrix to obtain image